

**MODIFIED MODEL OF INVESTMENT PORTFOLIO OPTIMIZATION
WHICH INCLUDES SYSTEMATIC RISKS LEVEL**

O. Voloshyna, T. Grygorova

Kremenchuk Mykhailo Ostrohradskyi National University

vul. Pershotravneva, 20, Kremenchuk, 39600, Ukraine. E-mail: voloksa@mail.ru; gasta1@yandex.ua

Aygul Auelbekova

L. N. Gumilyov Eurasian National University

vul. Satpayev, 2, Astana, 010000, Kazakhstan. E-mail: aigul.auelbekova@yandex.kz

Purpose. Standard deviation and practical use, beta-analysis and variation coefficient showed completely opposite results in some cases (based on Ukrainian stock market data). **Methodology.** We have made calculations of standard deviation index and beta coefficient for shares traded on the Ukrainian Stock Exchange. Profitability was calculated as price increase for 1 day. Ukrainian Stock Exchange index was taken as a base for Beta coefficient. In order to have accurate data, we took the same period in our study - one full year including non-working days. Calculations were made for 49 shares of Ukrainian companies. **Results.** Therefore, there is a need to learn the reasons of differences and develop an optimization model that will unite the various indicators. The study found out that standard deviation is deeply dependent on trade activity, and investment decisions in most cases are taken within a single market. In other words, investors have limited choice of instruments. The basis of standard deviation method is profitability, but it does not fit all investment strategies. Beta-coefficient method in addition takes into account the general tendencies of a particular financial market. Finally, variation coefficient allows to compare the risk with expected income. **Originality.** At falling markets, investors are increasingly using short positions instead of long and thereby falling market becomes profitable for them. If you change the approach to the calculation of return and consider a price decline as income instead of growth (purchasing tool operation is performed first, followed by sales), the results of calculations by model changes. Markowitz and Tobin models gave an error in the calculation of risk assessment. Therefore they can be considered ineffective for this strategy. The modified models of Markowitz gave the same result, but the portfolio was not diversified, because it included one tool, while the overall risk of the portfolio was 0 %, and the total yield was 0,8 %. **Practical value.** As a result, we believe that the developed model, based on standard deviation, beta coefficient, and variation coefficient, will provide investors with a one more tool while they develop their investment strategies.

Keys words: standard deviation, beta coefficient, investment portfolio, optimal portfolio, Markowitz model, the Tobin model.

**МОДИФІКОВАНА МОДЕЛЬ ОЦІНКИ ІНВЕСТИЦІЙНОГО ПОРТФЕЛЯ
З УРАХУВАННЯМ СИСТЕМАТИЧНОГО РИЗИКУ**

О. В. Волошина, Т. А. Григорова

Кременчуцький національний університет імені Михайла Остроградського

вул. Першотравнева, 20, м. Кременчук, 39600, Україна. E-mail: voloksa@mail.ru, gasta1@yandex.ua

А. К. Ауелбекова

Євразійський національний університет імені Л. Н. Гумільова

вул. Сатпаєв, 2, Астана, 010000, Казахстан. E-mail: aigul.auelbekova@yandex.kz

Фінансові ринки виконують як мінімум дві функції: забезпечення ресурсами господарюючих суб'єктів і надання можливостей заробітку інвесторам. І в тому, і в іншому випадках важливим є оцінка ризику. В роботі проведено оцінку кількісних показників ризику, що використовуються сучасною фінансовою наукою: стандартного відхилення, бета-коефіцієнтів та коефіцієнта варіації. Практичне використання цих показників продемонстрували абсолютно протилежні результати (на основі даних українського фондового ринку). У статті наведено причини відмінностей та розроблено модель оптимізації, яка об'єднує зазначені показники, мінімізуючи їх виявлені недоліки. Розроблена модель надасть інвесторам оптимальний набір інструментів в умовах різних інвестиційних стратегій.

Ключові слова: стандартне відхилення, бета-коефіцієнт, інвестиційний портфель, оптимальний портфель, модель Марковіца, модель Тобіна.

PROBLEM STATEMENT. Nowadays is forming the financial market of Ukraine. It has both - positive and negative sides. On one hand we can use the best practices of other countries, and it is positive side as it gives potential for growth. On the other hand - the financial market does not perform its basic function of supplying capital for extended payback, and this is negative. However, gradually the volume of transactions in the financial market is growing, as well as their role in the formation of GDP. If we look at the National

Commission on Securities and Stock statistics, we will see that at the end of 2010 total shares trade transactions market volume was 131,9 bln. UAH, while at the end of 2014 this number was 691,7 billion UAH. This means that market grew by more than 5 times. In comparison with national GDP, in 2010 Ukrainian stock market occupied 12,5 % while in 2014 it already stands for 39,55 % [1].

Let us point out that European vector for Ukraine opens new opportunities for financial investments, but it

also defines new challenges and tasks. The primary task is to adapt the mechanisms and management practices to European regulatory requirements. These mechanisms depend on who is an investor, what are his possibilities and how important it is to protect his interests.

If at the very beginning of Ukrainian financial market forming the major players were banks, nowadays the investors' structure is more diversified. Most securities at their nominal value are owned by legal entities – 432,71 billion UAH. A large proportion belongs to the state government – 231,35 bln UAH and to individuals (59,23 bln UAH), while banks own only 8,7 bln. UAH of securities nominal value out of 853 billion UAH in total [1].

Therefore, the role of private investors leads to the need to protect the investment of resources and reduce the risk of such investments.

Analysis the subject of the study. For a long time the world investment science has been taking into account not only income, which is possible to get, but also the risk that accompanies the investment process.

Quantitative risk assessment can significantly improve the effectiveness of financial investments management. The founder of the quantitative theory of risk assessment was Harry Markowitz. He started to use mathematical models to solve practical problems of investment. An important result of his research was portfolio theory, for which he received the Nobel Prize [2–3].

Briefly define the main points.

The essence of this theory is that the investor chooses among several investment alternatives only 2 important parameters: first is the expected rate of return (or asset's value growth percentage) and second is standard deviation of returns. Other words, the profitability is determined by the inclination of some midline that runs through the price of a financial instrument graph. The bigger the inclination, the greater is the profitability. And risk is determined as the actual price versus this midline: the greater the amplitude, the higher is the risk. Investors are seeking for higher return at lower level of risk. And higher risk is accepted only if profitability is significantly higher.

However, this is only a statistical evaluation of prices behavior in the past. And the main weakness of this theory is that it assumes that statistical evaluation allows to assess the level of these dimensions in the future. However, the different length of data will give quite different results. These drawbacks are observed in practice. Let us prove it by our example (Fig.1).

On (Fig. 1) you can see the dynamics share price «Donbasenergo». Thus, we see that for the period in 5 years, there are three completely different price behaviors, where the levels of mathematical income expectation and standard deviation are completely different. And these figures in entire period calculation will be completely not representative.



Figure 1 – Donbasenergo share price dynamics

However, the advantage of this theory is that the author suggests calculating parameters not by just individual instruments, but in general for the entire investment portfolio

Portfolio profitability is calculated as weighted average value:

$$r_p = \sum_{i=1}^n w_i r_i \quad (1)$$

And considering the level of connection between portfolio instruments, risk level for the portfolio will be calculated by the formula:

$$\sigma_p = \sqrt{w_i w_j \text{cov}_{ij}} = \sqrt{\sum_{i=1}^n w_i^2 \cdot \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=i+1}^n w_i \cdot w_j \cdot \text{corr}_{ij} \cdot \sigma_i \cdot \sigma_j} \quad (2)$$

where $\sigma_i \sigma_j$ – standard return's deviation of i, j -th financial instrument; corr_{ij} – correlation coefficient between i, j -th financial instrument; w_i – weight of i -th financial instrument (stocks) in portfolio; cov_{ij} – returns' covariation of i -th and j -th financial instruments; n – number of financial instruments in investment portfolio; r_i – return of i -th financial instrument.

This approach gave an opportunity for defining optimization tasks. The first is to determine such proportions between available financial instruments so that the risk is minimal within a given portfolio profitability.

$$\begin{cases} \sqrt{\sum_{i=1}^n w_i^2 \cdot \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=i+1}^n w_i \cdot w_j \cdot \text{corr}_{ij}} \rightarrow \min \\ \sum_{i=1}^n w_i r_i > r_p \\ \sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \end{cases} \quad (3)$$

The second reaches maximum return within a given level of risk.

$$\left\{ \begin{array}{l} \sum_{i=1}^n w_i r_i \rightarrow \max \\ \sqrt{\sum_{i=1}^n w_i^2 \cdot \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=i+1}^n w_i \cdot w_j \cdot \text{corr}_{ij} \cdot \sigma_i \cdot \sigma_j} < \sigma_p \quad .(4) \\ \sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \end{array} \right.$$

Considering the disadvantages, scientists continued to improve optimization portfolio mechanism.

This is how appeared the model of Tobin, which was Markowitz's theory continuation. Its difference is that the model introduced risk-free financial instruments, which are government debt securities [4].

Next model significantly differs from the above mentioned - Capital Asset Pricing Model, CAPM [5]. It is based on the fact that the future price of an asset is determined not by historical data, but as the sum of discounted cash flows. This model allows to determine the discount rate or profitability, which is a function of risk and not vice versa as in previous models. The risk in this case is divided into i) market or systematic, it cannot be diversified and ii) non-market or non-systematic - the risk of a separate financial instrument which can be diversified. Capital Asset Pricing Model is represented by the following function

$$E(r_i) = r_j + \beta_i [E(r_m) - r_f] = r_f + \beta_i E(r_m) - \beta_i r_f \quad .(5)$$

The value of beta is being used for market asset risk assessment. It shows the relationship between asset profitability and market profitability. Market profitability is a market portfolio rate of return, for which any index with a broad base can be taken. The value of beta is an angular coefficient of regression line inclination of asset return to the index. The theory of using beta coefficient has been widely used in practice.

We note that the development theory of portfolio investment is usually divided into three stages [6]. And the above mentioned theories belong to the second stage. But the development of Ukrainian and other financial markets in the former Soviet Union is so much lagging behind the global markets that it is too early to talk about the use of Black-Scholes options model. However, the direct copying of mentioned theories is also not possible in empirical researches.

The aim and results of study. The aim of our study is to justify the necessity for modification of existing investment portfolio optimization models and their adaptation to emerging markets.

As we can see, quantitative measurement of risk differs in two different theories. This difference can be quite significant, therefore it can fundamentally change investment decision. In order to check this hypothesis, we have made calculations of standard deviation index and beta coefficient for shares traded on the Ukrainian Stock Exchange. Profitability was calculated as price increase for 1 day. Ukrainian Stock Exchange index was taken as a base for Beta coefficient. In order to have accurate data, we took the same period in our study - one full year including

non-working days. Calculations were made for 49 shares of Ukrainian companies, which is close to 50. The number of 50 stocks was substantiated by western scientists, like J. Campbell, M. Lettau, I. Hu and B. Mal-kiel, who explored the dynamics of stock returns in 1960-1970 s and 1990 s and they came to the conclusion that modern portfolio should be quite diversified, as correlation between stocks declines and their volatility increases.

The results of rating for stocks based on their risk measured by the standard deviation are presented in table 1.

Table 1 – Rating of stock shares risk, measured by the standard deviation

Shares with relatively low risk		Shares with average risk		Shares with excessive risk	
Issuer	σ	Issuer	σ	Issuer	σ
CRTZ	1,10	KVBZ	3,20	CGOK	7,05
MSICH	1,43	INVBA	3,29	LUAZ	7,12
CEEN	1,70	ENMZ	3,36	ODEN	7,21
BAVL	2,07	SGOK	3,65	MTBD	7,22
UXUA	2,19	USCB	4,05	UTLM	7,49
DOEN	2,46	KRAZ	4,23	HMBZ	7,51
UNAF	2,63	NVTR	4,24	DNSS	7,53
AZST	2,99	AVDK	4,55	HRTR	7,54
		ZHEN	4,71	SMAS H	7,65
		ZAEN	5,61	LVON	7,71
		DNEN	5,68	KREN	8,19
		NITR	5,94	FLOT	8,32
		KEBO	6,09	STIR	8,49
		PHMS H	6,51	MMKI	8,90
		DNON	6,79	ALKZ	9,63
		KIEN	6,91	YASK	11,2
		FARM	6,92	SVGZ	11,3
		AZGM	6,99	SHKD	11,9
				LTPL	12,0
				KGLY K	12,7
				DOMZ	13,6
				SHCHZ	28,9
				MZVM	128,2

Thus, we have identified three groups. The first group includes stocks with relatively low risk. Stocks of 8 companies belong to it. Group's standard deviation is up to 3 %. The second is the group with average risk, it includes shares of 18 companies. Its standard deviation is up to 7 %. Given that majority of shares payback is less than 3 % and maximum is 8,2 %, this is quite significant risk. And the last group has 24 stocks with excessive risk at the level of 7 % - 128 %.

The results of stocks rating based on their risk measured by Beta-coefficient are presented in table 2.

Table 2 – Rating of stock shares risk, measured by Beta-coefficient

Issuer	β	Issuer	β	Issuer	β
DNON	-0,01	PHMSH	0,13	LUAZ	0,62
ZHEN	-0,01	ZAEN	0,16	MSICH	0,70
MMKI	-0,03	FLOT	0,17	DOMZ	-0,73
KEBO	0,04	ALKZ	0,23	STIR	0,75
CRTZ	0,07	CGOK	0,23	YASK	0,84
AZGM	0,08	KIEN	0,23	CEEN	0,87
HMBZ	-0,08	FARM	-0,24	HRTR	0,91
DNSS	0,09	UTLM	0,31	BAVL	0,95
UXUA	-0,09	KREN	0,34	SHKD	0,95
NVTR	0,10	SVGZ	0,42	DNEN	0,98
LVON	-0,10	SMASH	0,44	DOEN	1,04
KGLYK	-0,11	USCB	0,45	UNAF	1,06
SGOK	0,12	KRAZ	0,48	ENMZ	1,08
ODEN	0,12	NITR	-0,49	KVBZ	1,20
SHCHZ	0,13	LTPL	0,50	AZST	1,42
INVBA	0,13	MTBD	0,60	AVDK	1,58
				MZVM	1,59

If beta equals to 1, the stock has market average risk, if Beta is less than 1, the risk is lower than market average, and if Beta is above 1, the risk increases. As you can see, 3 out of 8 stocks with low risk have Beta coefficient above 1. Additional 3 have close to 1, and only 2 stocks have low level of non-systematic risk.

Thus the empirical study confirmed our hypothesis - two indicators give conflicting results. Therefore raises the question of rational use each of them.

The usefulness of risk measurement by standard deviation is reducing, because our own observations show that not all of the assumptions of the theory are followed in practice:

- First, since modern exchange trading is based on marginal trading, the level of investment resources can vary. Moreover, under unfavorable market conditions, investors are increasing the amount of investments, in order not to fix losses.
- Secondly, the significant deviation amplitude can be considered by investors as a way to generate income despite the risks.
- Thirdly, the sharper is the incline of the median price line, the more likely is the trend reversal. In other words the price incline changing to opposite will mean that the payback turns to opposite result - to losses:

Considering the disadvantages risk measurement in standard deviation and based on the fact that beta coefficient is one of the mandatory factors provided by the trade organizers which can be determined by both historical data and forecast, we suggested introducing this risk measurement into the optimization model.

If to analyze the results in calculations outlined on the slide 13, out of 50 stocks only 7 have higher than market average risk level. This is rather positive ratio and it indicates that the basis of investment risk level in emerging markets is to big extent not only issuer's risk, but also economic conditions: political stability, economic growth, lack of inflation, etc. Therefore, it reassures the need to consider this fact in the process of portfolio optimization.

Before making a modification model, we have made calculations by model Markowitz and Tobin based on data from the researched information array. Were calculated the following parameters needed to optimize the portfolio, variance, correlation coefficient, regression coefficient β , and covariance.

Table 3 – Results of investment portfolio optimization model based on Markovitz and Tobin theories

	Markovitz model	Tobin model
SHCHZ	76,75 %	
DOMZ	23,25 %	
Government bonds (risk-free)		93,12 %
MZVM		6,88 %
The over all risk of the portfolio	5,00 %	5,00 %
The total yield of the portfolio	2,63 %	7,38 %

As we can see both models formed different portfolios. However, each of them includes only two instruments. These portfolios should not be called as diversified. Therefore, it is also not possible to talk about avoiding risk through diversification.

According to Markowitz model, one of the tools occupies a significant share, almost 77 %, which creates additional risks. This is an increasing problem in the model of Tobin because one of the tools has a share of over 93 %. Since there must be present risk-free instruments in the Tobin model, it was added information about the yield of T-bills to an array. The high share of this instrument is quite predictable. The Ukrainian stock market is less active. The biggest turnovers are occurring exactly in government bonds.

This is partly due to administrative pressure from the central bank, but more - the lack of alternative instruments with sufficient liquidity. Tobin portfolio is more profitable at the same extent allowable risk on the level of 5 %.

Thus, the use of models in its pure form did not gave possibility to receive a diversified portfolio on the Ukrainian market.

As showed initial research of the beta coefficient the most actions have risk below the average, we proposed to use in assessing the risk the beta coefficient, which evaluates risk compared to the average instead of the standard deviation, which evaluates risk in its pure form.

Note that the realities in the emerging markets are such that any assets related to the risk, even the form of

commodities. This market can not offer a completely risk-free tool, so it becomes important the lack of excessive risk, but not absence of risk at all.

Stages of the models modification. Modification of the models was carried out in several stages. First beta coefficient was put into the Markowitz model as a limitation, given that the risk of the portfolio is calculated by the formula:

$$\sigma_p = \sqrt{\frac{\sigma_m}{n^2} \cdot \sum_{i=1}^n \frac{\beta_i}{corr_{im}} + \frac{n-1}{n} \overline{cov_{ij}}}, \quad (6)$$

where β_i – beta coefficient, a measure of the systematic risk of the i-th shares (market risk); $corr_{im}$ – yield correlation is the i-th asset (portfolio) with the yield of the market portfolio; σ_m^2 – the dispersion of

market returns; $\overline{cov_{ij}} = \frac{\sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n cov_{ij}}{n(n-1)}$ – average

covariance of returns of assets included in the portfolio.

After transformation model has acquired the following form:

$$\begin{cases} \sum_{i=1}^n w_i r_i \rightarrow \max \\ n^2 \sum_{i=1}^n w_i^2 \cdot \sigma_i^2 + 2n^2 \sum_{i=1}^n \sum_{j=i+1}^n w_i \cdot w_j \cdot corr_{ij} \cdot \sigma_i \cdot \sigma_j - \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n cov_{ij} < \sigma_m \sum_{i=1}^n \frac{\beta_i}{corr_{im}} \\ \sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \end{cases} \quad (7)$$

The corresponding modified model based on the model of Tobin is:

$$\begin{cases} w_0 r_0 + \sum_{i=1}^n w_i r_i \rightarrow \max \\ n^2 \sum_{i=1}^n w_i^2 \cdot \sigma_i^2 + 2n^2 \sum_{i=1}^n \sum_{j=i+1}^n w_i \cdot w_j \cdot corr_{ij} \cdot \sigma_i \cdot \sigma_j - \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n cov_{ij} < \sigma_m \sum_{i=1}^n \frac{\beta_i}{corr_{im}} \\ w_0 + \sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \end{cases} \quad (8)$$

Calculations have been conducted with a researched array of data were obtained following results are presented in table 4.

After analysis of the results we can say that a modification of models has led only to the refinement of risk with the influence of market trends and redistribution of tools shares in the portfolio. That is if used the presented model has not led to greater portfolio diversification in terms of its composition, but it gave a slightly lower concentration of one of the instruments in the portfolio (modified Markowitz model), which is positive. But opposite to Tobin modified model only slightly increased diversification.

Table 4 – Results of the optimization of the investment portfolio, modification 1

	Modified Markowitz model	Modified Tobin model
SHCHZ	63,3 %	
DOMZ	36,7 %	
Government bonds (risk-free)		94, 2%
MZVM		5,8 %
The over all risk of the portfolio	0,19 %	0,19 %
The total yield of the portfolio	2,46 %	7,37 %

For the further research it was proposed the models to consider risk in the form of beta portfolio which does not exceed 1. Thus modified model by Markowitz became looks like:

$$\begin{cases} \sum_{i=1}^n w_i r_i \rightarrow \max \\ \sum_{i=1}^n w_i \cdot \beta_i \leq 1 \\ \sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \end{cases} \quad (9)$$

Next was to adopt the model of Tobin. It is as follows:

$$\begin{cases} w_0 r_0 + \sum_{i=1}^n w_i r_i \rightarrow \max \\ \sum_{i=1}^n w_i \cdot \beta_i \leq 1 \\ w_0 + \sum_{i=1}^n w_i = 1 \\ w_i \geq 0 \end{cases} \quad (10)$$

The calculation results according to the models are shown in Table 5.

Table 5 – Results of the optimization of the investment portfolio, modification 2

	Modified Markowitz model	Modified Tobin model
DOMZ	25,35 %	
Government bonds (risk-free)		99,9 %
MZVM	74,65 %	0,1 %
The over all risk of the portfolio	1	1
The total yield of the portfolio	6,9 %	7,29 %

They yield of the portfolio increased from 2,63 % to 6,9 % due to change of tools (modified Markowitz model). A modified model of Tobin revealed that the portfolio became even less diversified. But one of the instruments in the portfolio coincided in both models, its appearance in a modified Markowitz model has led to significant growth of returns.

Therefore, we can conclude that the proposed model can be used for analysis of the market, excluding government bonds.

In the next step was considered the opposite strategy in financial operations. All previous models include yield increase with every next period so that is focused on the growing market. How to form the portfolio on falling market, which is largely developing markets? At such markets, investors are increasingly using short positions instead of long and thereby falling market becomes profitable for them. If you change the approach to the calculation of return and consider a price decline as income instead of growth (purchasing tool operation is performed first, followed by sales), the results of calculations by model changes.

Markowitz and Tobin models gave an error in the calculation of risk assessment. Therefore they can be considered ineffective for this strategy. The modified models of Markowitz gave the same result, but the portfolio was not diversified, because it included one tool, while the overall risk of the portfolio was 0 %, and the total yield was 0,8 %.

CONCLUSIONS. Thus, the use of mathematical models has helped us to evaluate the Ukrainian stock market. However, it is obtained only 2 of the 50 instruments worth investing. It can be concluded that the market is unsaturated and less active, so it is difficult to assess the possibility of estimated revenue on it. In general, we can note that the calculations showed once again the weakness of developing financial markets.

МОДИФИЦИРОВАННАЯ МОДЕЛЬ ОЦЕНКИ ИНВЕСТИЦИОННОГО ПОРТФЕЛЯ С УЧЕТОМ СИСТЕМАТИЧЕСКОГО РИСКА

О. В. Волошина, Т. А. Григорова

Кременчугский национальный университет имени Михаила Остроградского
ул. Первомайская, 20, г. Кременчуг, 39600, Украина. E-mail: voloksa@mail.ru, gasta1@yandex.ua

А. К. Ауелбекова

Евразийский национальный университет имени Л. Н. Гумилева
ул. Сатпаев, 2, Астана, 010000, Казахстан. E-mail: aigul.aelbekova@yandex.kz

Финансовые рынки призваны выполнить как минимум две функции: обеспечить ресурсами хозяйствующих субъектов и дать возможность заработать инвесторам. И в том, и другом случае важным является оценка риска. В работе произведена оценка количественных показателей риска, используемых современной финансовой наукой: стандартного отклонения, бета-коэффициентов и коэффициента вариации. Практическое использование этих показателей продемонстрировали совершенно противоположные результаты (на основе данных украинского фондового рынка). В статье приведены причины различий и разработана модель оптимизации, которая объединяет указанные показатели, минимизируя их выявленные недостатки. Разработанная модель, предоставит инвесторам оптимальный набор инструментов, в условиях разных инвестиционных стратегий.

Ключевые слова: стандартное отклонение, бета-коэффициент, инвестиционный портфель, оптимальный портфель, модель Марковица, модель Тобина.

During the research, we have grounded the necessity taking into account the Beta coefficient as a measure of non-systematic risk of the securities in the investment decisions. As a result, modification of the portfolio optimization model including beta coefficient led to maximize profitability by changing the instrument that was found in solving the optimization problem.

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