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DIFFERENTIAL TRANSFORM METHOD FOR SOLVING NON-LINEAR DIFFERENTIAL EQUATIONS BY THE ADOMIAN POLYNOMIALS

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Purpose. The aim of our study is to consider a novel approach for solving science and engineering problems are described by non-linear ordinary differential equations. **Results and originality.** The given approach consists in the joint use of the differential transform method and the method of Adomian polynomials. The main idea of proposed method based on solving equations in the image field with approximation of non-linear terms of differential equation by corresponding Adomian polynomials and henceforth obtaining of original of solution as a Taylor series. The proposed approach allows to overcome the mathematical obstacles during differential image calculation of complex non-linearities and essentially reduce the computational cost during searching of approximate solution of non-linear differential equation. **Practical value.** The proposed approach widens the application of differential transform method to deal with non-linear differential equations with different types of nonlinearity due to the properties and available algorithms of the Adomian polynomials. Numerical experiments demonstrate the application effectiveness of the proposed approach for solving non-linear differential equations and the good agreement with exact solution. References 18, figures 3, tables 3.

Key wards: nonlinear differential equation, differential transform method, polynomials Adomian, modified method

МЕТОД ДИФЕРЕНЦІАЛЬНИХ ПЕРЕТВОРЕНЬ ДЛЯ РОЗВ'ЯЗКУ НЕЛІНІЙНИХ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ЗА ДОПОМОГОЮ ПОЛІНОМІВ АДОМІАНА

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Розглянуто застосування модифікованого методу диференціальних перетворень для розв'язку задач, що описуються нелінійними диференціальними рівняннями. Метод засновано на сумісному використанні методу диференціальних перетворень та методу поліномів Адоміана. Основна ідея підходу полягає у розв'язку рівнянь в області зображень з апроксимацією нелінійних членів рівняння поліномами Адоміана та подальшим отриманням оригіналу розв'язку у вигляді ряду Тейлора. Запропонований підхід розширює застосування методу диференціальних перетворень до розв'язку нелінійних диференціальних рівнянь із складними типами нелінійності завдяки властивостям поліномів Адоміана. На відміну від традиційного методу диференціальних перетворень, застосування модифікованого методу диференціальних перетворень дозволяє уникнути математичних ускладнень, пов'язаних із складною нелінійністю диференціальних рівнянь, простіше у використанні та значно скорочує обсяг необхідних обчислень. Наведені приклади розв'язку диференціальних рівнянь з різними типами нелінійності (квадратична, логарифмічна та експоненціальна функція) показали ефективність застосування даного підходу та гарну збіжність з точним розв'язком

Ключові слова: нелінійне диференціальне рівняння, диференціальні перетворення, поліноми Адоміана, модифікований метод

PROBLEM STATEMENT. Non-linear differential equations often use for mathematical simulation of different problems in various fields of science and engineering. In general, non-linear differential equations don't have exact analytical solution and are solved by various numerical and approximate methods.

One of them is the numerical-analytical differential transform method (DTM) based on Taylor transformations was proposed by Pukhov G.E. and Zhou [1–3]. The main advantage of this method is that it can be applied directly to solve of non-linear differential equations without preliminary linearization, admits the possibility to obtain solution in analytic form and considerably reduces the volume of computation. The given method has found an effective application in various fields of science and engineering [4–9].

However, difficulties arise when it is applied to non-linear differential equations with complex nonlinearities. These difficulties can be overcoming by using of Adomian polynomials [10, 11]. The given approach allows the complex nonlinearities of differential equations to approximate by Adomian polynomials and obtain the solution of non-linear differential equation as a series, which terms are defined by known recurrence formulas for Adomian polynomials components.

Application of Adomian polynomials in the DTM (the modified differential transform method, MDTM) considerably simplifies solving non-linear differential equations and widens the field of DTM application.

The goal of this paper is to evaluate possibilities and application effectiveness of the modified differential transform method for solving problems described non-linear differential equations.

MATERIAL AND RESULTS. The DTM allows to replace in the mathematical model of a functional equation the functions $x(t)$ continuous argument t by their spectral models in the form of discrete functions $X(k)$ of integer argument $k = 0,1,2,..$

The differential transform of a given function $x(t)$ is defined as:

$$X(k) = \frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0}, \qquad (1)$$

where $x(t)$ is the original function, which represents the continuous and bounded with all its derivatives a function of real argument t, $X(k)$ is the discrete function of integer argument $k = 0,1,2,..$, which is termed as the differential image of original $x(t)$ (the differential spectrum); H is the scale stationary value having dimensionality t and often equals the time interval $0 \le t \le H$, over which we want to find the function $x(t)$.

The inverse differential transform, permitting by the image $X(k)$ to find the original $x(t)$ as a Taylor series, is introduced by:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t - t_0}{H} \right)^k X(k). \quad (2)$$

Hence

$$x(t) = \sum_{k=0}^{\infty} \frac{(t - t_0)^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=t_0}. \quad (3)$$

The value of H is to be less than the radius of convergence of series ρ, which can be determined on the base of D'Alembert criterion:

$$\rho = \overline{\lim_{k\to\infty}} \left| \frac{X(k)}{H^k} : \frac{H(k+1)}{H^{k+1}} \right| = H \overline{\lim_{k\to\infty}} \left| \frac{X(k)}{X(k+1)} \right|. \quad (4)$$

For particular cases at $t_0 = 0$, equations (2) and (3) will be following:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H} \right)^k X(k). \quad (5)$$

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0}. \quad (6)$$

*The method of Adomian polynomials***.** This approach uses the decomposition of non-linear differential equation into linear and non-linear parts and approximation of unknown non-linear part of equation by Adomian polynomials.

Consider the operator form of a following non-linear differential equation:

$$Px + Nx + Qx = c, \quad (7)$$

where $x = x(t)$; $P = \frac{d^n}{dt^n}$ is a non-linear differential operator, $n \succ 1$; $N = \frac{d}{dt}$ is a linear differential operator; Q represents the non-linear operator of non-linear function $f = f[x(t)]$, c is a source term**.**

According to the method of Adomian polynomials, the non-linear terms of equation are approximated by a series:

$$Qx = \sum_{m=0}^{\infty} A_m . \quad (8)$$

The solution $x(t)$ of required equation is defined as infinite series:

$$x(t) = \sum_{m=0}^{\infty} x_m(t). \quad (9)$$

Components of the Adomian polynomials are defined as:

$$A_m = \frac{1}{m!} \left\{ \frac{d^m}{d\lambda^m} \left[Q\left(\sum_{i=0}^{\infty} \lambda^i x_i\right) \right] \right\}_{\lambda=0} \quad (10)$$

The Adomian polynomials for non-linear function $f = f[x(t)]$ are determined into the form [12]:

$$\begin{aligned}
&A_0 = f(x_0),\ A_1 = x_1 f^{(1)}(x_0),\\
&A_2 = x_2 f^{(1)}(x_0) + \frac{1}{2!} x_1^2 f^{(2)}(x_0),\\
&A_3 = x_3 f^{(1)}(x_0) + x_1 x_2 f^{(2)}(x_0) + \frac{1}{3!} x_1^3 f^{(3)}(x_0),\\
&A_4 = x_4 f^{(1)}(x_0) + \left(x_1 x_3 + \frac{1}{2!} x_2^2 \right) f^{(2)}(x_0) +\\
&+ \frac{1}{2!} x_1^2 x_2 f^{(3)}(x_0) + \frac{1}{4!} x_1^4 f^{(4)}(x_0),\\
&A_5 = x_5 f^{(1)}(x_0) + (x_2 x_3 + x_1 x_4) f^{(2)}(x_0) +\\
&+ \frac{1}{2!}\left(x_1^2 x_3 + x_1 x_2^2\right) f^{(3)}(x_0) + \frac{1}{3!} x_1^3 x_2 f^{(4)}(x_0) +\\
&+ \frac{1}{5!} x_1^5 f^{(5)}(x_0), \ldots
\end{aligned} \quad (11)$$

The solution components $x_0, x_1, x_2 ....$ are defined using the following recurrence formulas:

$$x_0 = f,\ x_{k+1} = -P^{-1} R x_k - P^{-1} A_k, \quad k \ge 0. \quad (12)$$

In [13], the effective algorithm for the Adomian polynomials calculation using only operations of addition and multiplication has been proposed:

$$\begin{aligned}
&A_0 = f(x_0),\\
&A_n = \sum_{k=1}^{n} C_n^k f^{(k)}(x_0),\ n \ge 1,
\end{aligned} \quad (13)$$

where

$$\begin{aligned}
&C_n^1 = x_n,\ n \ge 1,\\
&C_n^k = \frac{1}{n} \sum_{j=0}^{n-k} (j+1) x_{j+1} C_{n-1-j}^{k-1},\ 2 \le k \le n.
\end{aligned} \quad (14)$$

*The modified DTM***.** Taking into account the features of the DTM, components of differential image of non-linear function $f[x(t)]$ of required differential equation at $t_0 = 0$ are defined in the following form [14]:

$$
\begin{aligned}
&F(0) = f(x(0)) = f(X(0)) = f(x_0),\\
&F(1) = \frac{d}{dt} f(x(t))\Big|_{t=0} = x'(0) f^{(1)}(x(0)) = X(1) f^{(1)}(X(0)),\\
&F(2) = X(2) f^{(1)}(X(0)) + \frac{1}{2!}(X(1))^2 f^{(2)}(X(0)),\\
&F(3) = X(3) f^{(1)}(X(0)) + X(1)X(2) f^{(2)}(X(0)) + \frac{1}{3!}(X(1))^3 f^{(3)}(X(0)),\\
&F(4) = X(4) f^{(1)}(X(0)) + (X(1)X(3) + \frac{1}{2!}(X(2))^2) f^{(2)}(X(0)) + \frac{1}{2!}(X(1))^2 X(2) f^{(3)}(X(0)) +\\
&+ \frac{1}{4!}(X(1))^4 f^{(4)}(X(0)),\\
&F(5) = X(5) f^{(1)}(X(0)) + (X(2)X(3) + X(1)X(4)) f^{(2)}(X(0)) + \frac{1}{2!}(X(1))^2 X(3) + X(1)(X(2))^2) f^{(3)}(X(0)) +\\
&+ \frac{1}{3!}(X(1))^3 X(2) f^{(4)}(X(0)) + \frac{1}{5!}(X(1))^5 f^{(5)}(X(0)), \ldots
\end{aligned}
\qquad (15)
$$

Comparison equations (11) and (15) shows that components of differential image of original of non-linear function of differential equation are similar in mathematical form to the corresponding components of Adomian polynomials. So, we can take, that components of differential image of original of non-linear function of required equation can be obtained from corresponded components of Adomian polynomials by replacing each solution components $x_k(t)$ by the corresponding component of differential image $X(k)$ of the same index.

In [14] shown, that such replacing can be applied for any types of nonlinearity of differential equations. Therefore, for solving non-linear differential equations we can applied the combined method of differential transformations with approximation of non-linear term of equation by Adomian polynomials using the following algorithm. We construct the spectral model of required differential equation. In this model, the differential image of original of non-linear function $F(k)$ replaces by components $\tilde{A}_k$, which are obtained from components A_k of Adomian polynomials by replacing each components x_k by the corresponding component of differential image $X(k)$ of the same index k. Then we calculate discretes of differential image of equation and, accounting (2) or (5), obtain the original of solution of required differential equation.

Accounting the existence of effective methods of the Adomian polynomials calculation, such approach allows to overcome the mathematical obstacles during differential image calculation of complex non-linearities and essentially reduces the computational cost during searching of approximate solution of non-linear differential equation.

Numerical experiments. The effectiveness of MDTM application for solving non-linear ordinary differential equations can be illustrated by following examples.

Example 1. Let us consider the non-linear differential equation with the quadrated source term [15]:

$$\frac{dx(t)}{dt} = 2x(t) - x^2(t) + 1, \quad x(0) = 0. \qquad (16)$$

The exact solution is given by:

$$x(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right).$$

By applying the DTM, we write the equation (16) in the spectral form:

$$
\begin{aligned}
&(k+1)X(k+1) = 2X(k) - \tilde{A}_k + \sigma(k),\\
&X(0) = 0,
\end{aligned}
\qquad (17)
$$

where $\sigma(k) = \begin{cases} 1, & k = 0 \\ 0, & k \ge 1. \end{cases}$

Accordingly, with procedure (11), for non-linear part of equation (16) $f[x(t)] = x^2(t)$ we calculate the component A_k of Adomian polynomials and thereon the corresponding components $\tilde{A}_k$ for replacement by them of components of differential images of non-linear part of equation:

$\tilde{A}_0 = X^2(0)$, $\tilde{A}_1 = 2X(0)X(1)$,

$\tilde{A}_2 = X^2(1) + 2X(0)X(2)$,

$\tilde{A}_3 = 2X(0)X(3) + 2X(1)X(2)$,

$\tilde{A}_4 = 2X(0)X(4) + 2X(1)X(3) + X^2(2)$,

$\tilde{A}_5 = 2X(0)X(5) + 2(X(2)X(3) + X(1)X(4))$

Substituting values in (17) we obtain the corresponding differential discretes:

$$X(0) = 0,\ X(1) = 1,\ X(2) = 1,\ X(3) = 1/3,$$

$$X(4) = -1/3,\ X(5) = -7/15,\ X(6) = -7/45, \ldots$$

Hence, with taking into account (5), the approximate solution of equation (16) at $H = 1$ will be following:

$$x(t) = t + t^2 + \frac{1}{3}t^3 - \frac{1}{3}t^4 - \frac{7}{15}t^5 - \frac{7}{45}t^6 + \ldots \qquad (18)$$

The given solution is the series expansion in Taylor polynomials of exact solution:

$$x(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right).$$

Fig. 1 shows a comparison between exact solution and approximate solution by the MDTM. In Table 1 shows the relative error and the approximate solution by the MDTM, which was obtained using first 5 discretes of differential spectra of the differential equation (16).

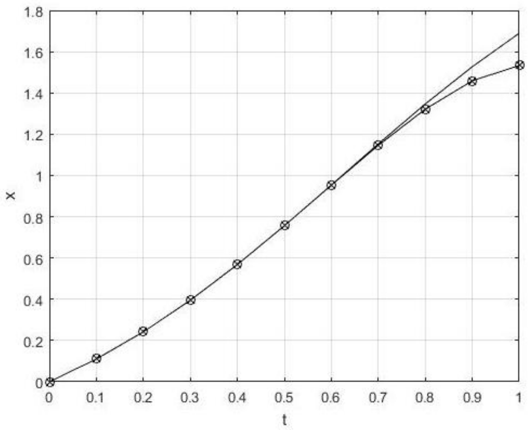

Figure 1 – Comparison of exact solution (-) and approximate solution by the MDTM (⊗) for example 1

Table 1 – Numerical results of Example 1

t	Exact solution	MDTM	Relative error, ε_r
0	0	0	0
0,1	0,110295	0,110295	8,07e-08
0,2	0,241977	0,241984	4,26e-06
0,3	0,395105	0,395166	3,62e-05
0,4	0,567812	0,568021	1,24e-04
0,5	0,756014	0,756250	1,39e-04
0,6	0,953566	0,952512	6,24e-04
0,7	1,152949	1,145867	4,19e-03
0,8	1,346364	1,321216	1,49e-02
0,9	1,526911	1,458738	4,04e-02
1,0	1,689498	1,533333	9,24e-02

Example 2. Let us consider the non-linear differential equation with initial conditions [14, 16]:

$$\frac{d^2x(t)}{dt^2} = 2x(t) + 4x(t)\cdot \ln x(t), \quad x(t) > 0 . \qquad (19)$$
$$x(0) = 1, \ \dot{x}(1) = 0$$

The exact solution is given by: $x(t) = e^{t^2}$.

The spectral model of equation (19) is defined as:

$$(k+1)(k+2)X(k+2) = 2X(k) + \tilde{A}_k , . \qquad (20)$$
$$X(0) = 1, \ \ X(1) = 0$$

Accordingly, with procedure (11), for non-linear part of equation (19) $f\left[x(t)\right] = 4x(t)\ln x(t)$ we calculate the component A_k of Adomian polynomials and corresponding components $\tilde{A}_k$ for replacement by them of components of differential images:

$$\tilde{A}_0 = 4X(0)\ln X(0), \ \tilde{A}_1 = 4X(1)(\ln X(0)+1),$$
$$\tilde{A}_2 = 4X(2)(\ln X(0)+1) + 2X^2(1)X^{-1}(0),$$
$$\tilde{A}_3 = 4X(3)(\ln X(0)+1) + 4X(1)X(2)X^{-1}(0) -$$
$$-\frac{4}{3!}X^3(1)X^{-2}(0),$$
$$\tilde{A}_4 = 4X(4)(\ln X(0)+1) + 4X^{-1}(0)\cdot(X(1)X(3) +$$
$$+\frac{1}{2!}X^2(2)) - \frac{4}{2!}X^2(1)X(2)X^{-2}(0) + \frac{8}{4!}X^4(1)X^{-3}(0),$$
$$\tilde{A}_5 = 4X(5)(\ln X(0)+1) + 4X^{-1}(0)\cdot(X(2)X(3) +$$
$$+X(1)X(4)) - \frac{4}{2!}X^{-2}(0)\cdot(X^2(1)X(3) + X(1)X^2(2)) +$$
$$+\frac{8}{3!}X^3(1)X(2)X^{-3}(0) - \frac{24}{5!}X^5(1)X^{-4}(0), \ldots$$

Substituting values of $\tilde{A}_k$ in (20) we obtain the corresponding discretes of differential image:

$$X(0) = 1, \ X(1) = 0, \ X(2) = 1, \ X(3) = 0,$$
$$X(4) = \frac{1}{2!}, \ X(5) = 0, \ X(6) = \frac{1}{3!}, \ X(7) = 0, \qquad (21)$$
$$X(8) = \frac{1}{4!}, \ldots$$

Taking into account (5), the approximate solution of required equation (19) at $H = 1$ will be following:

$$x(t) = 1 + t^2 + \frac{1}{2!}t^4 + \frac{1}{3!}t^6 + \frac{1}{4!}t^8 + \ldots =$$
$$= \sum_{k=0}^{\infty} \frac{1}{k!}\left[t^2\right]^k = e^{t^2}. \qquad (22)$$

Fig. 2 shows a comparison between exact solution and approximate solution by the MDTM. In Table 2 shows the relative error and the approximate solution by the MDTM, which was obtained using first 5 discretes of differential spectra of the differential equation (19).

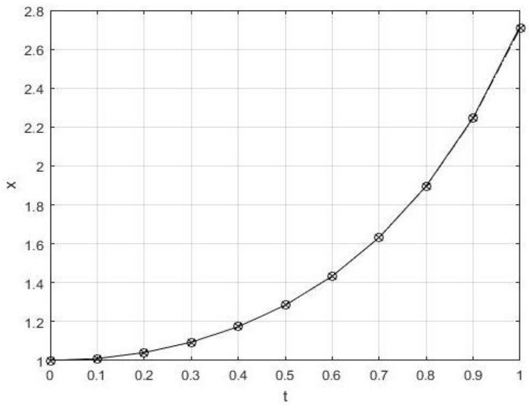

Figure 2 – Comparison of exact solution (-) and approximate solution by the MDTM (⊗) for example 2

Table 2 – Numerical results of Example 2

t	**Exact solution**	**MDTM**	**Relative error, ε_r**
0	1	1	0
0,1	1,010050	1,010050	3,07e-13
0,2	1,040810	1,040811	3,16e-10
0,3	1,094174	1,094174	1,84e-08
0,4	1,173511	1,173510	3,30e-07
0,5	1,284025	1,284017	3,12e-06
0,6	1,433329	1,433276	1,97e-05
0,7	1,632316	1,632060	9,42e-05
0,8	1,896481	1,895481	3,68e-04
0,9	2,247908	2,244560	1,23e-03
1,0	2,718282	2,708333	3,66e-03

Example 3. Let us consider the non-linear differential equation with exponential source term [14, 17]:

$$\frac{d^2x(t)}{dt^2} = 2e^{x(t)}, \quad 0 < t < 1, \tag{23}$$

$$x(0) = 0, \ \dot{x}(0) = 0$$

The exact solution is given by: $x(t) = -2\ln(\cos t)$

For MDTM application, we present the spectral model of equation (23) as:

$$\begin{aligned}&(k+1)(k+2)X(k+2) = 2\tilde{A}_k \\ &X(0) = 0, \ X(1) = 0\end{aligned}. \tag{24}$$

Accordingly, with procedure (11), for non-linear part of equation (23) $f[x(t)] = 2e^{x(t)}$ we calculate the component A_k of Adomian polynomials and corresponding components $\tilde{A}_k$ for replacement by them of corresponding components of differential images:

$$\tilde{A}_0 = e^{X(0)}, \ \tilde{A}_1 = X(1)e^{X(0)},$$

$$\tilde{A}_2 = \left(X(2) + \frac{1}{2!}X^2(1)\right)e^{X(0)},$$

$$\tilde{A}_3 = \left(X(3) + X(1)X(2) + \frac{1}{3!}X^3(1)\right)e^{X(0)},$$

$$\tilde{A}_4 = \begin{pmatrix} X(4) + X(1)X(3) + \frac{1}{2!}X^2(2) + \\ + \frac{1}{2!}X^2(1)X(2) + \frac{1}{4!}X^4(1) \end{pmatrix} e^{X(0)},$$

$$\tilde{A}_5 = \begin{pmatrix} X(5) + X(2)X(3) + X(1)X(4) + \\ + \frac{1}{2!}\left(X^2(1)X(3) + X(1)X^2(2)\right) + \\ + \frac{1}{3!}X^3(1)X(2) + \frac{1}{5!}X^5(1) \end{pmatrix} e^{X(0)}.$$

Substituting values of $\tilde{A}_k$ in (24) we obtain the following discretes of differential image of required equation:

$$X(0) = 0, \ X(1) = 0, \ X(2) = 1, \ X(3) = 0,$$

$$X(4) = \frac{1}{6}, \ X(5) = 0, \ X(6) = \frac{2}{45}, \ X(7) = 0,$$

$$X(8) = \frac{17}{1260}, \ldots.$$

Taking into account (5), the solution of required equation (19) at $H = 1$ will be following:

$$y(x) = x^2 + \frac{1}{6}x^4 + \frac{2}{45}x^6 + \frac{17}{1260}x^8 + \ldots, \tag{25}$$

Fig. 3 shows a comparison between exact solution and approximate solution by the MDTM. In Table 3 shows the relative error and the approximate solution by the MDTM, which was obtained using first 5 discretes of differential spectra of the differential equation (23).

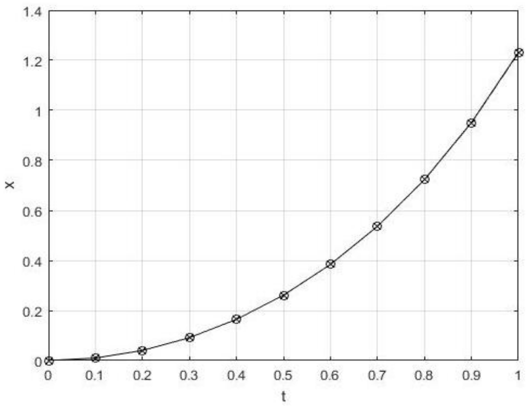

Figure 3 – Comparison of exact solution (-) and approximate solution by the MDTM (⊗) for example 3

Table 3 – Numerical results of Example 3

t	Exact solution	MDTM	Relative error, ε_r
0	0	0	0
0,1	0,010017	0,010017	1,29e-15
0,2	0,040270	0,040270	4,98e-12
0,3	0,091383	0,091383	6,58e-10
0,4	0,164458	0,164458	2,13e-08
0,5	0,261168	0,261168	3,21e-07
0,6	0,383930	0,383927	2,99e-06
0,7	0,536172	0,536147	2,00e-05
0,8	0,722781	0,722651	1,06e-04
0,9	0,950885	0,950303	4,73e-04
1,0	1,231253	1,228977	1,85e-03

Numerical experiments on the MDTM application for solving non-linear differential equations in partial derivatives also have shown the good agreement with exact solution [18].

CONCLUSIONS. The MDTM application for solving non-linear differential equations which are described various problems of science and engineering was reviewed. The method is based on solving differential equation in the image field with approximation of non-linear term of equation by Adomian polynomials and further obtaining of original of solution as a Taylor series. The solution examples of differential equations with different types of nonlinearity (quadratic, logogriphic and exponential function) are presented. The obtained results shown the good agreement with exact solution. Compared with traditional DTM, the MDTM allows to overcome the mathematical obstacles, related to complex non-linearity of differential equations, simpler in application and significantly reduces the computation volume.

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МЕТОД ДИФФЕРЕНЦИАЛЬНЫХ ПРЕОБРАЗОВАНИЙ ДЛЯ РЕШЕНИЯ НЕЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ПОМОЩЬЮ ПОЛИНОМОВ АДОМИАНА

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Рассмотрено применение модифицированного метода дифференциальных преобразований для решения задач, описываемых нелинейными дифференциальными уравнениями. Метод основан на решении уравнений в области изображений с аппроксимацией нелинейных членов уравнения полиномами Адомиана и дальнейшим получением оригинала решения в виде ряда Тейлора. Предложенный подход расширяет применение метода дифференциальных преобразований к решению нелинейных дифференциальных уравнений со сложными типами нелинейности благодаря свойствам полиномов Адомиана. В отличие от традиционного метода дифференциальных преобразований, применение модифицированного метода дифференциальных преобразований позволяет избежать математических сложностей, связанных со сложной нелинейностью дифференциальных уравнений, проще в использовании и значительно сокращает объем необходимых вычислений. Приведенные примеры решения дифференциальных уравнений с разными типами нелинейности (квадратичная, логарифмическая и экспоненциальная функция) показали эффективность применения данного подхода и хорошую сходимость с точным решением.

Ключевые слова: нелинейное дифференциальное уравнение, дифференциальные преобразования, полиномы Адомиана, модифицированный метод.

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