UDC 539.3:517.518.12+624.044 DOI https://doi.org/10.32782/1995-0519.2023.1.2

# MATHEMATICAL MODELING OF THE STRESS-STRAIN STATE OF A PLATE WITH RIGID LINEAR INCLUSION AND MIXED BOUNDARY CONDITIONS

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Purpose of this work is to develop an effective mathematical model for determining the stress-strain state of the plate under bending with rigid linear inclusion, mixed boundary conditions and its numerical implementation. Methodology. A rectangular plate is considered, with two opposite edges simply supported, and the other two edges clamped. To reduce the boundary value problem of bending this plate to the singular integral equation, the methods of generalized integral transformations and the construction of the Green function were used. The identified nature of the singularity of the resulting integral equation was taken into account. To obtain an approximate solution, the method of orthogonal polynomials is used. Findings. A mathematical model is constructed and an approximate solution of the specified boundary point problem is obtained. As a result, the values determining the deflections of the inclusion and the distribution of the deflections of the plate at different values of the inclusion lengths and stiffness characteristics of the plate were calculated. Originality. As is known, there is no single approach to solving this class of problems. The method presented in the article is based on the reduction of a boundary value problem to a singular integral equation. Determining the nature of the singularity of the equation made it possible to reduce it to a rapidly converging system of linear algebraic equations, which indicates a high efficiency of the method. The mathematical model of this boundary value problem and the results of the numerical-analytical method make it possible to apply them to the solution of the corresponding class of problems, to determine the amount of deformation of the structure of plates with inclusion under given boundary conditions and to predict their optimal parameters for design or manufacture. Practical value and conclusions. The deformed state of the plate and the deflection of the inclusion depending on the geometric and stiffening parameters are investigated. According to the final formulas, numerical calculations were performed, the results of which are presented in the table and graphs. Practical importance has the proposed method and numerical results. For a particular case, the results obtained were compared with those previously known. A slight difference (about 0.01%) indicates a high accuracy of the results of calculations on the proposed model.

**Key words:** stress-strain state, boundary value problem, plate, bending, generalized transverse force, stress concentrator, inclusion, Green's function, singular integral equation.

**Problem statement.** Determining the stress-strain state (SSS) of materials with inclusions, reinforcements, and other inhomogeneities is an important task due to the rapid development of new composite materials, their growing use in various industries, including the military, and the lack of a unified approach to solving such problems. It is known that the presence of inhomogeneities in structural elements significantly complicates the calculation of SSS, because inhomogeneities are stress concentrators and significantly affect the strength of these structures. Therefore, the method of modeling SSS of plate with stress concentrator under load considered in this article is actual both in theoretical and practical aspects.

It is also known that in some cases of the determination of stress-strain state (SSS) of constructions it is possible to obtain an accurate analytical solution for the corresponding mathematical models. So, for example, in [1; 2] an algorithm for constructing a closed analytical solution to the problem of oscillations of the beam, the width of which varies according to a special law, is given. Based on a combination of factorization and symmetry methods in combination with the approximation of the corresponding functions, an approach is defined by which a solution can be obtained. Namely, to find the frequency equations, hence the natural frequencies, and construct the Eigen forms of oscillations for the beam of a given configuration.

But for most boundary problems of applied mechanics, it is possible to determine the SSS of constructions only with the help of approximate solution methods.

One of the methods for solving many classes of boundary value problems of mechanics is the numerical-analytical method of boundary elements or one of its varieties – the method of integral transformations [3–8]. It does not require the availability of large capacities for high-performance computing on a computer. This method is based on reducing the boundary problem to an integral equation or a system of integral equations and their subsequent solution. In the case of a singularity of integral equations, the nature of this singularity is taken into account. With the help of this method, it is possible to obtain both effective approximate (numerical) and exact solutions to boundary value problems. Namely. In workse [3], the method was used to solve the problems of bending plates with linear inhomogeneities oriented arbitrarily. The application of singular integral equations to the solution of a number of problems with technological defects in fracture mechanics is considered in the works [4; 5]. Fourier-Laplace integral transformations and a method for constructing discontinuous functions were also used. In the works [6; 7] the main analytical dependencies of the numerical-analytical method of boundary elements in relation to the calculation of bendable orthotropic plates with the subsequent implementation of the algorithm are obtained. In [8] the method of solving the problems of bending cylindrical gentle shells with thin rigid linear inclusions was further developed and detailed. Namely, the method of integral transformations and orthogonal polynomials obtained numerical solutions of boundary value problems about the bending of a simply supported cylindrical sloping shell with an inclusion parallel to the prime meridian.

In this article, we consider the problem of plate bending -a/2≤x,y≤a/2 with a thin rigid inclusion on the segment y=0, c/2≤x≤c/2 (c<a). The plate is clamped along the two edges y=±a/2, -a/2≤x≤/2 and simply supported on the edges x=±a/2; -a/2≤y≤a/2. The bending of the plates occurs due to the action on the inclusion of the vertical force P at the point x=y=0. It is necessary to find the relationship between the value of the force P and the vertical displacement of inclusion $W_0$ and the distribution of the deflections of the plate W(x,y).

**Material and results.** The mathematical formulation of the problem is carried out according to the scheme of works [3, 8, 9] and will consist in finding unknown W(x,y), $W_0$ and $\Psi(x)$), satisfying the equation:

$$D\Delta^2 W(x,y) = \Psi(x)\delta(y),\ -a/2\le x,y \le a/2, \quad (1)$$

boundary conditions

$$x=\pm\frac{a}{2}:\ W(x,y)=\frac{\partial^2 W(x,y)}{\partial x^2}=0, \quad (2)$$

$$y=\pm\frac{a}{2}:\ W(x,y)=\frac{\partial W(x,y)}{\partial x}=0, \quad (3)$$

condition on inclusion

$$W(x,0)=W_0,\ -c/2\le x\le c/2\ (c<a) \quad (4)$$

and equilibrium condition of inclusion

$$P=\int_{-\frac{c}{2}}^{+\frac{c}{2}}\Psi(x)\,dx,\ -c/2\le x\le c/2\ (c<a) \quad (5)$$

Here D is the flexural rigidity of the plate, $\delta(y)$ – Dirac delta function

$\Psi(x)=V_y(x_1-0)-V_y(x_1+0)$ – the jumps of the generalized shearing force during the transition through the inclusion,

$$V_y(x,y) = -D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial y \partial x^2}\right],$$

$\nu$ – Poisson's coefficient.

To solve the problem, apply to (1), (3) Fourier transform on variable x:

$$W_\alpha(y) = \int_{-\frac{a}{2}}^{\frac{a}{2}} W(x,y)\cos\alpha x dx\ ;$$

$$W(x,y) = \frac{2}{a}\sum_{n=1,3,5}^{\infty} W_\alpha(y)\cos\alpha x\ ;\ \alpha = \frac{\pi n}{a}$$

Taking into account (2), the result is

$$\left(\frac{d^2}{dy^2} - \alpha^2\right)^2 w_\alpha(y) = \frac{\delta(y)}{D}\Psi_\alpha \qquad (6)$$

$$y=\pm\frac{a}{2}\ ;\ w_\alpha(y) = w'_\alpha(y) = 0 \qquad (7)$$

In here $\Psi_\alpha$ - Fourier transformation of function $\Psi(x)$.

Green's function $G_\alpha(y,\eta)$ of boundary value problem (6) – (7) we build according to the scheme of work [8]. As a result, we come to the following:

$$G_\alpha(y,\eta) = \text{¦}\,(y-\eta) - \sum_{i=0}^{3}\text{¨}_i(y)U_i(\eta)\ ; \qquad (8)$$

$$\Phi(y-\eta) = \frac{1+\alpha|y-\eta|}{4\alpha^3}\exp(-\alpha|y-\eta|);$$

$$\begin{Bmatrix} U_0(\eta) \\ U_1(\eta) \end{Bmatrix} = \frac{1+\alpha\left|\frac{b}{2}\pm\eta\right|}{4\alpha^3}\exp\left(-\alpha\left|\frac{b}{2}\pm\eta\right|\right);$$

$$\begin{Bmatrix} U_2(\eta) \\ U_3(\eta) \end{Bmatrix} = \pm\frac{\left(\frac{b}{2}+\eta\right)\exp(-\alpha\left|\frac{b}{2}\pm\eta\right|}{4\alpha};$$

$$\text{¨}_0(y) = \varphi_2(y) - \varphi_1(y)\ ;\ \text{¨}_1(y) = \varphi_2(y) + \varphi_1(y);$$

$$\varphi_1(y) = \frac{(ch\rho + \rho sh\rho)sh(\alpha y) - \alpha y ch\rho ch\alpha y}{2(sh\rho ch\rho - \rho)};$$

$$\varphi_2(y)\frac{(sh\rho + \rho ch\rho)ch\alpha y - \alpha y sh\rho sh\alpha y}{2(sh\rho ch\rho + \rho)};$$

$$\text{¨}_2(y) = \varphi_1^*(y) - \varphi_2^*(y)\ ;\ \text{¨}_3(y) = \varphi_1^*(y) + \varphi_2^*(y);$$

$$\varphi_1^*(y) = \frac{\frac{b}{2}ch\rho sh\alpha y - y sh\rho ch\alpha y}{2(\rho - sh\rho ch\rho)};$$

$$\varphi_2^*(y) = \frac{-\frac{b}{2}sh\rho ch\alpha y + y ch\rho sh\alpha y}{2(\rho + sh\rho ch\rho)}\ ;\ \rho = \frac{\alpha a}{2} = \frac{\pi n}{2};$$

Thus, the solution of the problem (6) – (7), taking into account (8), will be recorded as follows:

$$W_\alpha(y) = \frac{\text{¨}_\alpha}{D}G_\alpha(y,0)\ ; \qquad (9)$$

Hence

$$W(x,y) = \frac{2}{aD}\int_{-\frac{c}{2}}^{\frac{c}{2}}\sum_{n=1,3,5}^{\infty}\text{¨}(t)cos\alpha t G_\alpha(y,0)cos\alpha x dt\ ; \qquad (10)$$

Realizing the condition on inclusion (4), we come to the integral equation

$$\frac{2}{aD}\int_{-\frac{c}{2}}^{\frac{c}{2}}\text{¨}(t)\left(\sum_{n=1,3,5}^{\infty}G_\alpha(0,0)cos\alpha t cos x\right)dt = W_0\ ;\ -\frac{c}{2} \le x \le \frac{c}{2}.$$

Here

$$G_\alpha(0,0) = \frac{a^3}{4\pi^3 n^3}\chi_n\ ;\ \chi_n = 1 - \frac{sh\rho + \rho ch\rho}{sh\rho ch\rho + \rho}(1+\rho)e^{-\rho} - \frac{\rho^2 sh\rho e^{-\rho}}{\rho + sh\rho ch\rho}$$

Entering notation

$$x = t\frac{c}{2}\ ;\ t = \tau\frac{c}{2}\ ;\ \text{¨}(t) = \frac{2}{c}\text{¨}_1(\tau)\ ;\ \varepsilon = \frac{c}{a}\ ;\ \gamma = \frac{\pi n\varepsilon}{2}\ ; \qquad (11)$$

and given that the function $\Psi_1(\tau)$ is even, come to the following equation

$$\frac{a^2}{2\pi^3}D\int_{-1}^{1}\text{¨}_1(\tau)\left(\sum_{n=1,3,5}^{\infty}\frac{\cos\gamma t\cos\gamma\tau}{n^3}\chi_n\right)d\tau = W_0\ ;\ (-1 \le t \le 1) \qquad (12)$$

Since $\chi_n > 0, and\ \chi_n \to 1\ when\ n \to \infty$, then, on the basis of these asymptotic estimates and by analogy with the works [8; 9] we conclude that equation (12) has no solutions in the class of functions with integrable features at the ends of ±1, so we look for solution (12) in the form of the next series

$$\Psi_1(\tau) = \sum_{k=0}^{\infty}\Psi_k P_k(\tau) \qquad (13)$$

where

$$P_0(\tau) = \frac{1}{\sqrt{1-\tau^2}}\ ;\ P_n(\tau) = \frac{2\sqrt{\pi}(2n)!P_{2n}^{-\frac{3}{2},-\frac{3}{2}}(\tau)}{\Gamma\left(2n-\frac{1}{2}\right)(1-\tau^2)^{\frac{3}{2}}}\ (n \ge 1),$$

$P_n^{\alpha,\beta}(\tau)$ – Jacobi polynomials $\Psi_\kappa$ – unknown constants. Using the method of orthogonal polynomials according to the scheme of work [8], we obtain an infinite system of linear algebraic equations with respect to unknown $\Psi_k$.

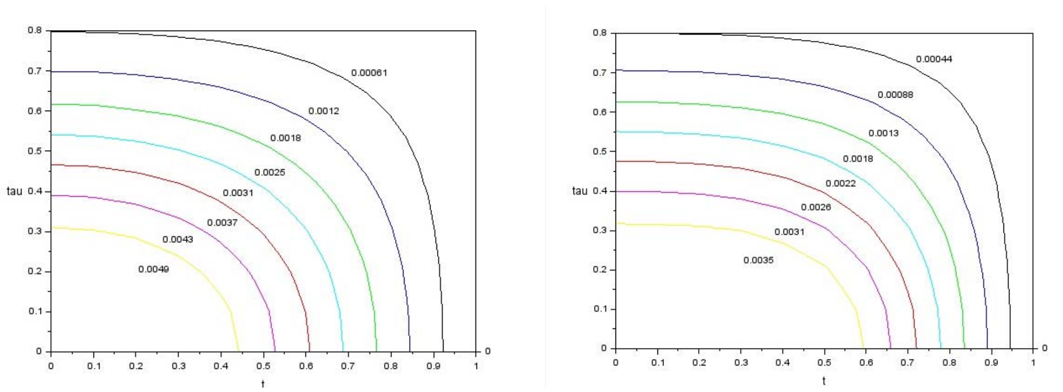
$$\sum_{k=0}^{\infty}C_{mk}\text{¨}_k = f_m\ ;\ m = 0,\infty \qquad (14)$$

$$C_{0,0} = \sum_{n=1,3,5}^{\infty}\frac{[J_0(\gamma)]^2}{n^3}\chi_n\ ;\ C_{0,k} = C_{k,0} = (-1)^k\sum_{n=1,3,5}^{\infty}\gamma\frac{J_0(g)J_{2k-1}(\gamma)}{n^3};$$

$$C_{m,k} = (-1)^{m+k}\sum_{n=1,3,5}^{\infty}\gamma^2\frac{J_{2m-1}(\gamma)J_{2k-1}(\gamma)}{n}\ ;\ f_m = 2\pi^2 DW_0 a^{-2}\delta_{om};$$

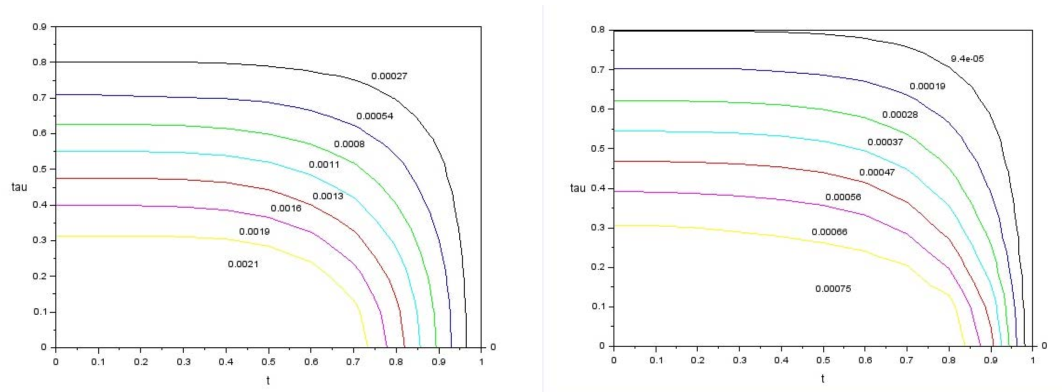
($\delta_{0m}$ – Kronecker symbol, $J_n(x)$ – Bessel function of the 1-st kind)

Using the equilibrium condition (5), we obtain that $\Psi_0 = \frac{P}{\pi}$. Hence we find the desired relationship




ε = 0.2ε = 0.4

Fig. 1. Contour plot of $W^*$ at $\varepsilon = 0.2$, $\varepsilon = 0.4$.




ε = 0.6ε = 0.8

Fig. 2. Contour plot of $W^*$ at $\varepsilon = 0.6$, $\varepsilon = 0.8$.

Table 1

**Values of $\alpha_1$ depending on the size $\mu$**

| ε | 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha_1$ | 7,04 | 6,01 | 5,02 | 4,08 | 3,21 | 2,39 | 1,61 | 1,02 | 0,57 | 0,19 |

between the forces P and displacement of the inclusion $W_0$:

$$W_0 = \frac{Pa^2}{D}\alpha(\varepsilon,\nu); \qquad (15)$$

$\alpha = \frac{1}{\pi\Psi^*_{0}}$; $\Psi^*_\tau\left(\tau = \overline{0,\infty}\right)$ – coefficients in decomposition (13) to be sought when $f_m = 2\pi^2\delta_{om}$ in (14).

The deflections of the plates are found using (10). By entering the symbols (11) in (10) and presenting the solution as a series (13), we finally obtain:

$$W\left(\frac{at}{2},\frac{b\tau}{2}\right) = \frac{Pa^2}{D}W^*(t,\tau,\nu,\varepsilon); \qquad (16)$$

where

$$W^* = \frac{1}{2\pi^3}\sum_{n=1,3,5}^{\infty}\frac{g_n(\tau)}{n^3}\cos\frac{\pi n t}{2}\,\omega_n;$$

$$\omega_n = J_0(\gamma) + \gamma\sum_{\tau=1}^{\infty}\frac{\ddot{\tau}^*}{\ddot{0}^*}(-1)^\tau J_{2\tau-1}(\gamma); \quad q_n(\tau) = q_1(\tau) - \sum_{i=2}^{5} q_i(\tau);$$

$$q_1(\tau) = (1+\rho|\tau|)\exp(-\rho|\tau|);$$

$$q_{2,3}(\tau) = (1+\rho)\exp(-\rho)\left(\widetilde{\varphi_2}(\tau) \mp \widetilde{\varphi_1}(\tau)\right);$$

$$\widetilde{\varphi_1}(\tau) = \frac{(ch\rho + \rho sh\rho) - \rho\tau ch\rho ch\rho\tau}{2(sh\rho ch\rho - \rho)}; \quad \widetilde{\varphi_2}(\tau) = \frac{(sh\rho + \rho ch\rho)ch\rho\tau - \rho\tau sh\rho sh\rho\tau}{2(sh\rho ch\rho + \rho)};$$

$$q_{4,5}(\tau) = \pm\exp(-\rho)\left(\widetilde{\varphi_1^*}(\tau) \mp \widetilde{\varphi_2^*}(\tau)\right);$$

$$\widetilde{\varphi_1^*}(\tau) = \frac{\rho^2\left[ch\rho sh\rho\tau - \tau sh\rho ch\rho\tau\right]}{2(\rho - sh\rho ch\rho)}; \quad \widetilde{\varphi_2^*}(\tau) = \frac{\rho^2\left[-sh\rho ch\rho\tau + \tau ch\rho sh\rho\tau\right]}{2(\rho + sh\rho ch\rho)}.$$

Displacement (deflection) of inclusion, as shown above, is presented as $W_0 = \frac{Pa^2}{D}\alpha(\varepsilon,\nu)$, plate deflections: $W\left(\frac{at}{2},\frac{b\tau}{2}\right) = \frac{Pa^2}{D}W^*(t,\tau,\nu,\varepsilon)$ (see formulas (15), (16)). According to these formulas, the values

$\alpha_1 = \alpha(\varepsilon, 0.3)10^3; W^*(t, \tau, 0.3, \varepsilon)$, were calculated. These values determine the deflection of the inclusion and deflection of the plate at different values of the lengths of inclusion (from 0 to 0.9). Values $\alpha_1$ for different values $\varepsilon$ are presented in table 1. Figures 1, 2 show the plots of level values $W^*$ at $\varepsilon = 0.2$, $\varepsilon = 0.4$, $\varepsilon = 0.6$, $\varepsilon = 0.8$. Figures 1, 2 show the notation tau = $\tau$.

**Conclusions.** At $\varepsilon = 0$, that is for the particular case of our problem, the obtained results $\alpha_1$ were compared with previously known ones [10]. The difference between the calculation results by the two methods did not exceed 0.1%. Such a slight error indicates the high accuracy of the calculation results for the proposed mathematical model. The results of the calculations are shown in the graphs (Fig. 1,2). Sufficient accuracy of calculations of 0.001 is obtained while maintaining five – six equations in the system (14), which shows the high efficiency of this method of solving the boundary value problem. Practical importance has the proposed method and numerical results, which allow predicting the amount of deformation of the structure of plates with inclusions and determining their optimal parameters for design or manufacture.

## REFERENCES

1. Trapezon, K.O. (2021). Uzahalnenyi metod symetrii dlia zadachi pro pozdovzhni chy krutylni kolyvannia stryzhni v zminnoi zhorstkosti [Generalized method of symmetrie for the problem about longitudinal or torsional vibrations of variable rigid rods]. Visnik KrNU imeni Mykhaila Ostrogradskogo – Bulletin of Kremenchuk Mykhailo Ostrohradskyi National University, 2(127), 94–99 [in Ukranian]. doi.org/10.30929/1995-0519.2021.2.94–99.

2. Trapezon, K.O., & Trapezon, O.G. (2021). Vlasni zghynni kolyvannia balky zi spetsialnym zakonom zminy shyryny [Natural bending vibrations of the beam with the special law of change of width]. Visnik KrNU imeni Mykhaila Ostrogradskogo – Bulletin of Kremenchuk Mykhailo Ostrohradskyi National University, 4(129), 116–123 [in Ukranian]. doi: 10.30929/1995–0519.2021.4. 116-123.

3. Papkovskaya, O.B., Kozin, A.B., & Kamara, D. (2008). Matematicheskaya model izgiba ortotropnoi plastini s krivolineinoi proizvolno orientirovannoi neodnorodnostyu [Mathematical model of bending of the orthotropic plate with curvilinear arbitrarily oriented heterogeneity]. Trudy Odesskogo Polytechescskogo Universiteta – Proceedings of Odessa Polytechnic University, 1(29), 237–241 [in Russian].

4. Usov, A.V., & Batyrev A.A. (2010). Matematicheskoe modelirovanie protsessov kontrolya pokritii elementov konstruktsii na baze singulyarnykh integralnykh uravnenii [Mathematical modeling of control processes of coatings of structural elements based on singular integral equations]. Problemy mashinostroenija – Journal of Mechanical Engineering, 13(1), 65–75 [in Russian].

5. Usov, A.V., Kunitsyn, M.V., Klimenko, D.V., & Davidyuk, V.M. (2022). Modeliuvannia vplyvu stokhastychnykh defektiv, shcho utvoriuiutsia u vyrobakh pid chas mekhanichnoi obrobky, na vtratu yikh funktsionalnykh zalezhnostei [Modeling the impact of stochastic defects formed in products during mechanical processing on the loss of their functional dependencies]. Pratsi Odeskoho Politekhnychnoho Universytetu – Proceedings of Odessa Polytechnic University, 1(65), 16–29 [in Ukranian].

6. Suryaninov, N.G., & Pavlenko, I.V. (2014). Prilozhenie chislenno-analiticheskogo metoda granichnykh elementov k raschetu ortotropnykh plastin [Application of the numerical analytical method of boundary elements to the calculation of orthotropic plates]. Pratsi Odeskoho Politekhnychnoho Universytetu – Proceedings of Odessa Polytechnic University. 1(43), 18–27 [in Russian].

7. Suryaninov, N.G., Pavlenko, I.V., & Shotadze, G.B. (2015). Matematicheskaia model ortotropnoi plastiny na osnove metoda granichnyh elementov [Mathematical model of an orthotropic plate based on the boundary element method]. Visnik Kiiivskogo Natsional'nogo Universitetu Technologii ta Dyzajnu – Bulletin of the Kyiv National University of Technology and Design. Ser. : Tehnichni nauki, 3, 50–56 [in Russian].

8. Kozin, O.B., & Papkovskaya, O.B. (2016). Analysis of stress-strain state of the shell with the inclusion in bending. Pratsi Odeskoho Politekhnychnoho Universytetu – Proceedings of Odessa Polytechnic University. 1(48), 24–29.

9. Kozin, O.B., Papkovskaya, O.B., & Kozina, M.O. (2016). Modeling and solution of contact problem for infinite plate and cross–shaped embedment. Pratsi Odeskoho Politekhnychnoho Universytetu – Proceedings of Odessa Polytechnic University. 2(49), 97–103.

10. Popov, G.Ya. (2007). Izbrannye Trudy [Selected Works]. Odessa : VMV, vol 2 [in Russian].

# МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ПЛАСТИНИ З ЖОРСТКИМ ЛІНІЙНИМ ВКЛЮЧЕННЯМ ТА ЗМІШАНИМИ ГРАНИЧНИМИ УМОВАМИ

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Метою даної є розробка ефективної математичної моделі визначення напружено – деформованого стану пластини при вигині з жорстким лінійним включенням, змішаними граничними умовами та її чисельна реалізація. Розглядається прямокутна пластина, два протилежні краї якої вільно оперті, а два інші защемлені. Для зведення крайової задачі згину цієї пластини до сингулярного інтегрального рівняння застосовувалися методи узагальнених інтегральних перетворень та побудови функції Гріна. Враховувався виявлений характер сингулярності одержаного інтегрального рівняння. Для отримання наближеного рішення використано метод ортогональних полиномів. Побудовано математичну модель та отримано наближене вирішення зазначеної крайової задачі. В результаті були обчислені величини, що визначають прогини включення та розподіл прогинів пластини при різних значеннях довжин включення та жорсткосних характеристик пластини. Як відомо, єдиного підходу до вирішення цього класу задач не існує. Викладений у статті метод базується на зведенні крайової задачі до сингулярного інтегрального рівняння. Визначення характеру сингулярності рівняння дозволило звести його до системи лінійних алгебраїчних рівнянь, що швидко збігається, що свідчить про високу ефективність метода. Математична модель даної крайової задачі та результати чисельно – аналітичного методу дозволяють застосовувати їх до вирішення відповідного класу задач, визначати величину деформації конструкції пластин із включенням за даних граничних умов та прогнозувати їх оптимальні параметри для проектування чи виготовлення. За підсумковими формулами були виконані чисельні розрахунки, результати яких відображені у таблиці та графіках. Для окремого випадку отримані результати були порівняні з раніше відомими. Незначна різниця (близько 0.01%) свідчить про високу точність результатів розрахунків за запропонованою моделлю.

**Ключові слова:** напружено-деформований стан, крайова задача, пластина, вигин, узагальнена поперечна сила, концентратор напруг, включення, функція Гріна, сінгулярне інтегральне рівняння.

## ЛІТЕРАТУРА

1. Трапезон К.О. Узагальнений метод симетрій для задачі про поздовжні чи крутильні коливання стрижні в змінної жорсткості. *Вісник* КрНУ імені Михайла Остроградського. 2021. Вип. 2. С. 94–99. doi:https://doi.org/10.30929/1995-0519.2021.2.94-99.

2. Трапезон К.О., Трапезон К.О. Власні згинні коливання балки зі спеціальним законом зміни ширини. *Вісник* КрНУ імені Михайла Остроградського. 2021. Вип. 4. С. 116–123. doi: 10.30929/1995-0519.2021.4.116–123.

3. Папковская О.Б., Козин А.Б., Камара Д. Математическая модель изгиба ортотропной пластины с криволинейной произвольно ориентированной неоднородностью. Труды Одесского политехнического университета. 2008. Вып. 1(29). С. 237–241.

4. Усов, А.В., Батырев А.А. Математическое моделирование процессов контроля покрытий элементов конструкций на базе сингулярных интегральных уравнений. Проблемы машиностроения. 2010. Т. 13. № 1. С. 65–75.

5. Усов, А.В., Куніцин М.В., Клименко Д.В., Давидюк В.М. Моделювання впливу стохастичних дефектів, що утворюються у виробах під час механічної обробки, на втрату їх функціональних залежностей. Праці Одеського політехничного ун-ту. 2022. Вип. 1(65). С. 16–29.

6. Сурьянинов Н.Г., Павленко И.В. Приложение численно-аналитического метода граничных элементов

к расчету ортотропных пластин. Праці Одеського політехничного ун-ту. 2014. Вип. 1(43). С. 18–27.

7. Сурьянинов Н.Г., Павленко И.В., Шотадзе Г.Б. Математическая модель ортотропной пластины на основе метода граничных элементов. *Вісник* Київського національного університету технологій та дизайну. Серія «Технічні науки». 2015. № 3. С. 50–56.

8. Kozin O.B., Papkovskaya O. B. Analysis of stress-strain state of the shell with the inclusion in bending. Праці Одеського політехничного ун-ту. 2016. Вип. 1(48). С. 24–29.

9. Kozin O.B., Papkovskaya O.B., Kozina M.O. Modeling and solution of contact problem for infinite plate and cross-shaped embedment. Праці Одеського політехничного ун-ту. 2016. Вип. 2(49). С. 97–103.

10. Попов Г.Я. Избранные труды. Одесса : ВМВ, 2007. Т. 2. 504 с.

*Стаття надійшла 07.02.2023*